Closing Thurs: 3.1-2
Closing Fri: 3.3
Closing next Fri: 3.4(1), 3.4(2)

Exam 1 is Tuesday, Oct 24th in quiz section. 2.1-2.3,2.5-2.8, 3.1-3.3.

- One 8.5 by 11 inch sheet of ***handwritten*** notes (front and back)
- A Ti-30x IIs calculator (no other calc)
- Pen or pencil (no red or green)
- No make-up exams.

All homework is fair game. Know the concepts well. Practice on old exams.

6. $\frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + f'(x)g(x)$

7. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

Entry Task: Find the derivatives of

1) $y = (x^4 + 3)^2 + x^5 e^x$

2) $y = \frac{2x^2 + 1}{x^3 e^x}$

$\boxed{1}$ $y = x^8 + 6x^4 + 9 + x^5 e^x$

$$\frac{dy}{dx} = 8x^7 + 24x^3 + 0 + x^5 e^x + 5x^4 e^x$$
$$= 8x^7 + 24x^3 + e^x x^4 (x+5)$$

$\boxed{2}$

$$y' = \frac{x^3 e^x (4x) - (2x^2+1)(x^3 e^x + 3x^2 e^x)}{(x^3 e^x)^2}$$

$$y' = \frac{4x^4 e^x - x^2 e^x (2x^2+1)(x+3)}{x^6 e^{2x}}$$

$$y' = \frac{x^2 e^x (4x^2 - (2x^2+1)(x+3))}{x^6 e^{2x}}$$

$$y' = \frac{4x^2 - (2x^2+1)(x+3)}{x^4 e^x}$$

"Proof" of product rule

$$\lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x+h)g(x)+f(x+h)g(x)-f(x)g(x)}{h}$$

$$=\lim_{h\to 0}f(x+h)\underbrace{\frac{g(x+h)-g(x)}{h}}_{g'(x)}+g(x)\underbrace{\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}}_{f'(x)}$$

$$f(x)\quad g'(x)\quad + \quad g(x)\quad f'(x)$$

3.3 Derivatives of Trig Functions

First a review: you will need to know all the following well in Math 124/5/6.

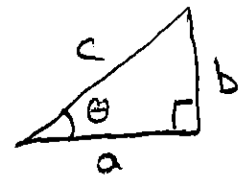
1. Triangle definitions

$\sin(x) = \dfrac{\text{opp}}{\text{hyp}}$	$\cos(x) = \dfrac{\text{adj}}{\text{hyp}}$
$\tan(x) = \dfrac{\text{opp}}{\text{adj}}$	$\cot(x) = \dfrac{\text{adj}}{\text{opp}}$
$\sec(x) = \dfrac{\text{hyp}}{\text{adj}}$	$\csc(x) = \dfrac{\text{hyp}}{\text{opp}}$

Thus,

$\sec(x) = \dfrac{1}{\cos(x)}$	$\csc(x) = \dfrac{1}{\sin(x)}$
$\tan(x) = \dfrac{\sin(x)}{\cos(x)}$	$\cot(\text{x}) = \dfrac{\cos(x)}{\sin(x)}$

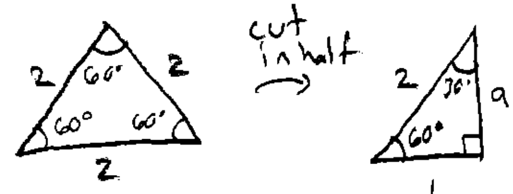
2. **Know what their graphs look like**.
3. **Know their inverses** and how to use them (and how to get more solutions)



$\sin\theta = \frac{b}{c}$

$\tan\theta = \frac{b}{a}$

$\csc\theta = \frac{c}{b}$



THUS 60°

$\cos(\pi/3) = \frac{1}{2}$

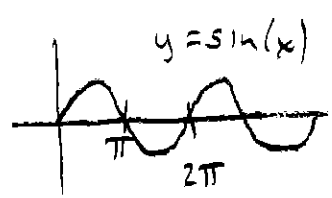
$\sin(\pi/3) = \frac{\sqrt{3}}{2}$

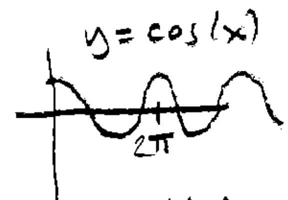
$\tan(\pi/3) = \sqrt{3}$

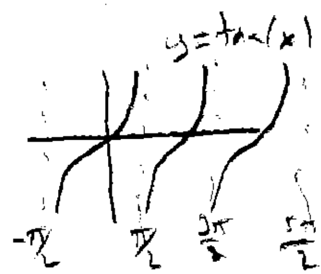
$1^2 + a^2 = 2^2$

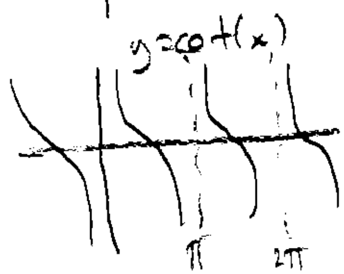
$a^2 = 3$

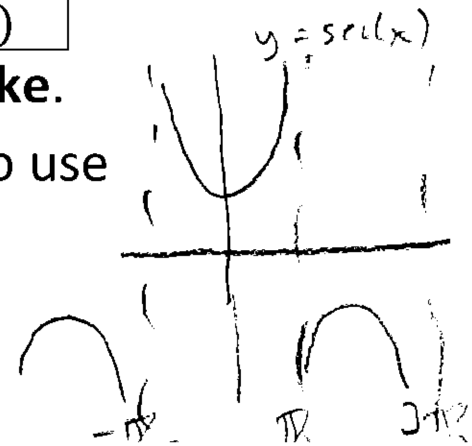
$a = \sqrt{3}$

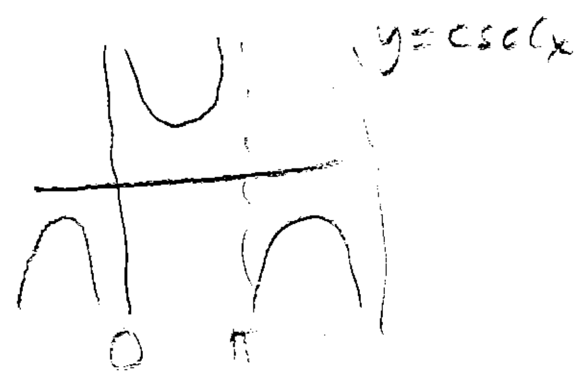








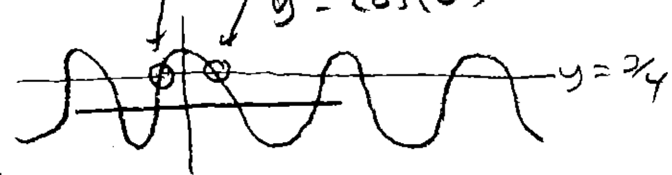




FIND ALL SOL'NS TO $\cos(\theta) = 3/4$

PRINCIPAL SOL'N: $\theta = \cos^{-1}(3/4) \approx 0.722734$ radians

SYMMETRIC SOL'N: $\theta = -\cos^{-1}(3/4) = -0.722734$



ALL SOL'NS:

$$\boxed{\begin{aligned} \theta &= \cos^{-1}(3/4) + 2\pi k \\ \theta &= -\cos^{-1}(3/4) + 2\pi k \end{aligned}}$$ for all integers k

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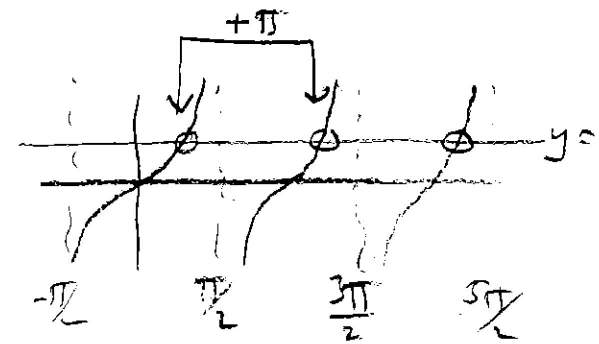
FIND ALL SOL'NS TO $\tan(5t) = 1$



FIRST SOLVE $\tan(\theta) = 1$

PRINCIPAL SOL'N: $\theta = \tan^{-1}(1) = \pi/4$

ALL SOL'NS: $\theta = \pi/4 + \pi k$ for any integer k

$\to \theta = \ldots, \pi/4, \pi/4 + \pi, \pi/4 + 2\pi, \pi/4 + 3\pi, \ldots$

REPLACE θ WITH $5t$

$\Rightarrow 5t = \ldots, \pi/4, \pi/4 + \pi, \ldots$

$t = \ldots, \pi/20, \frac{\pi}{20} + \frac{\pi}{5}, \pi/20 + \frac{2\pi}{5}, \ldots$

$$\boxed{t = \frac{\pi}{20} + \frac{\pi}{5}k \quad \text{for any integer } k}$$

4. Know the standard values (unit circle) and circular motion

Examples (do NOT use a calculator)

$$\cos\left(\frac{\pi}{6}\right) = \sqrt{3}/2$$

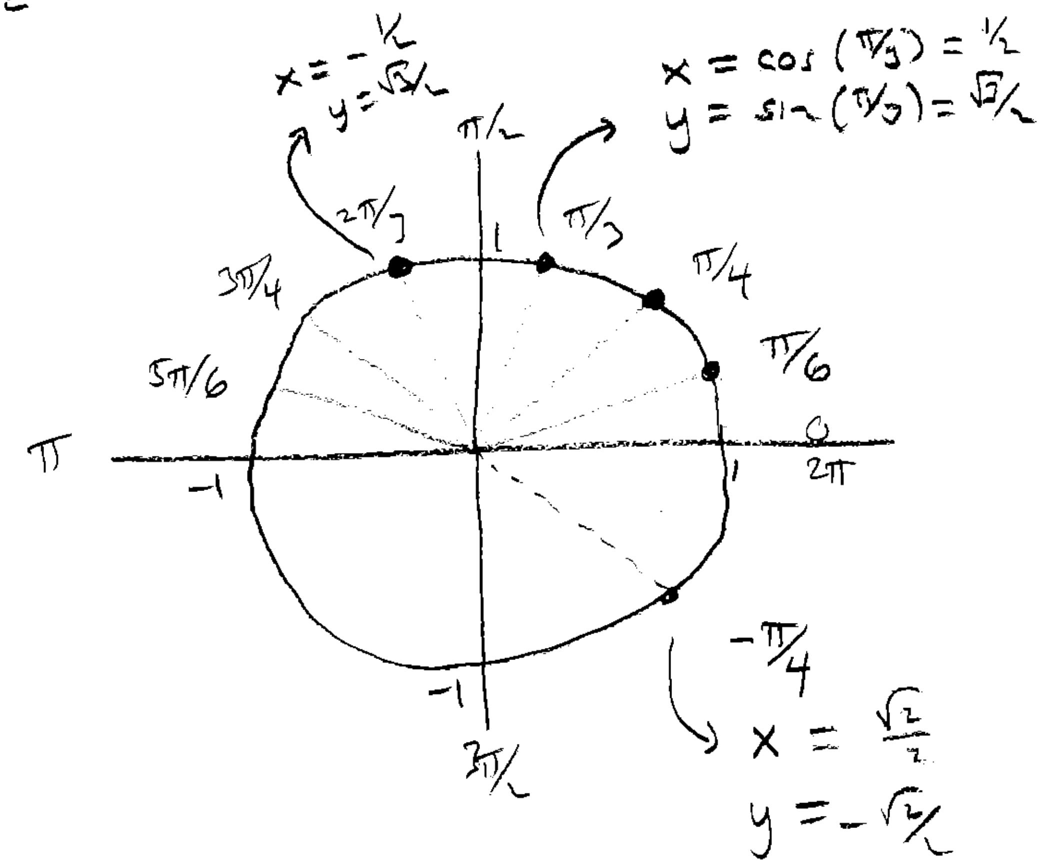
$$\sec\left(-\frac{\pi}{4}\right) = \frac{1}{\cos(-\pi/4)} = \frac{1}{\sqrt{2}/2} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$\tan\left(\frac{2\pi}{3}\right) = \frac{\sin(2\pi/3)}{\cos(2\pi/3)} = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$$

$$\sin^{-1}\left(\frac{1}{2}\right) = \pi/6$$

$$\tan^{-1}(1) = \pi/4$$

	Θ	$\cos(\theta)$	$\sin\theta$
$0°$	0	1	0
$30°$	$\pi/6$	$\sqrt{3}/2$	$1/2$
$45°$	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$60°$	$\pi/3$	$1/2$	$\sqrt{3}/2$
$90°$	$\pi/2$	0	1



5. Know the main identities.

$$\sin^2(x) + \cos^2(x) = 1$$

$$2\sin(x)\cos(x) = \sin(2x)$$

For lecture today, we also need:

$\sin(a+b) = \sin(a)\cos(b) + \cos(\mathrm{a})\sin(b)$

$\cos(a+b) = \cos(\mathrm{a})\cos(b) - \sin(a)\sin(b)$

Consider $f(x) = \sin(x)$.

Then

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cos(x)\sin(h)}{h} + \frac{\sin(x)\cos(h) - \sin(x)}{h}$$

$$= \cos(x) \underbrace{\lim_{h\to 0} \frac{\sin(h)}{h}}_{*} + \sin(x) \underbrace{\lim_{h\to 0} \frac{\cos(h) - 1}{h}}_{**} = \cos(x)\cdot 1 + \sin(x)\cdot 0$$

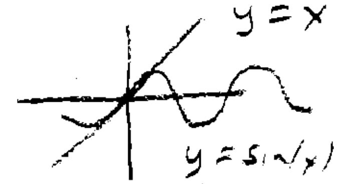
$$= \cos(x)$$

THUS, $\boxed{\frac{d}{dx}(\sin(x)) = \cos(x)}$.

$$* \lim_{h\to 0} \frac{\sin(h)}{h} = 1$$

SEE MY POSTED PROOF

ONLY TRUE IN RADIANS!

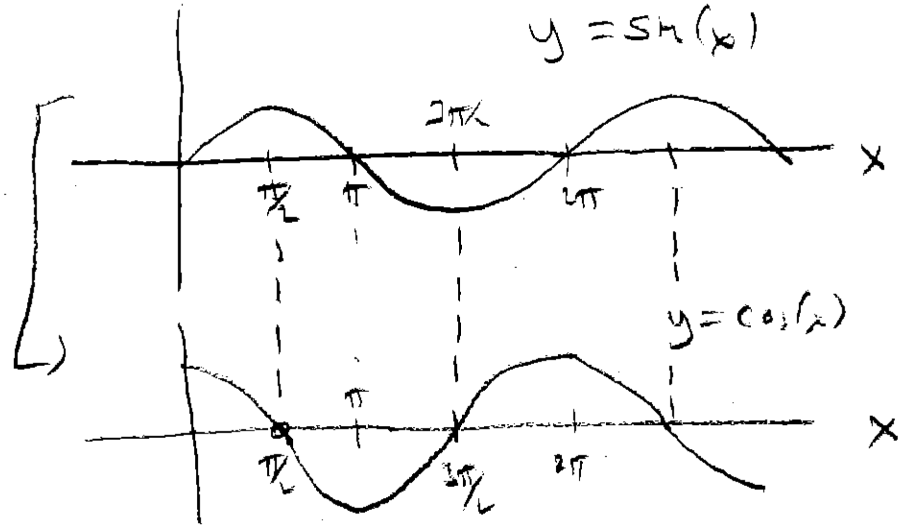


as $x \approx 0$

$\sin(x) \approx x$

$$** \lim_{h\to 0} \frac{(\cos(h) - 1)}{h} = 0$$

SEE PROOF IN BOOK



ALSO $\frac{d}{dx}\left(\cos(x)\right) = -\sin(x)$

Ex)

$$\frac{d}{dx}\left(\tan(x)\right)$$

$$= \frac{d}{dx}\left(\frac{\sin(x)}{\cos(x)}\right) \begin{matrix} \leftarrow N \\ \leftarrow D \end{matrix}$$

$$= \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$$

$$= \frac{1}{\cos^2(x)} = \sec^2(x)$$

OTHER TRIG

Ex)

$$\sec(x) = \frac{1}{\cos(x)} \begin{matrix} \leftarrow N \\ \leftarrow D \end{matrix}$$

$$\frac{d}{dx}\left(\sec(x)\right) = \frac{\cos(x)\cdot 0 - 1(-\sin(x))}{\cos^2(x)}$$

$$= \frac{\sin(x)}{\cos^2(x)} = \frac{1}{\cos(x)}\,\frac{\sin(x)}{\cos(x)}$$

$$= \sec(x)\tan(x)$$

NOTE: $\lim\limits_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$

$$\lim_{t\to 0} \frac{\sin(5t)}{5t} = 1$$

$$\lim_{t\to 0} \frac{\sin(5t)}{t} = \lim_{t\to 0} \frac{5\sin(5t)}{5t}$$

$$= 5$$

$\frac{d}{dx}(\sin(x)) = \cos(x)$	$\frac{d}{dx}(\cos(x)) = -\sin(x)$
$\frac{d}{dx}(\tan(x)) = \sec^2(x)$	$\frac{d}{dx}(\cot(x)) = -\csc^2(x)$
$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$	$\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$

COMBINING WITH OTHER RULES

Ex) $y = x^2\cos(x)$

$$y' = x^2(-\sin(x)) + 2x\cos(x)$$
$$= -x^2\sin(x) + 2x\cos(x)$$
$$= x(-x\sin(x) + 2\cos(x))$$

Ex) $y = \dfrac{\tan(x)}{x^3}$ ← N, ← D

$$y' = \frac{x^3\sec^2(x) - 3x^2\tan(x)}{x^6}$$

$$y' = \frac{x^2(x\sec^2(x) - 3\tan(x))}{x^6}$$

$$y' = \frac{x\sec^2(x) - 3\tan(x)}{x^4}$$